# Longitudinal Computational Modeling

## 1 Introduction

Across multiple assessment strategies, self-report measure scores, behavioral performance, or neural processes may be reflected by overall aggregate indices. For these measures, a critical challenge is that individual behavioral outputs are produced via multiple psychological processes (Wiecki et al., 2015). Thus, observable output behavior represents a gross measure of many competing and complementary processes. While conventional scoring procedures are unable to discriminate between these processes, more recently developed generative models of behavior are well positioned to discriminate between individual psychological processes, yielding enhanced specificity in behavioral metrics as well as improved tasks psychometrics and better clarity for probing associations with psychological processes (Ahn et al., 2017; Chen et al., 2015; Huys et al., 2016).The use of generative modeling is becoming commonplace in multiple fields of study, with the large majority of studies relying on cross-sectional designs. There is a growing number of studies that have begun examining test-retest reliability across a small number of assessment waves. However, for applications of generative modeling to understand longer-term change across multiple contexts, including naturalistic course or in the context of intervention, more than two assessment occasions are needed. Longitudinal changes are frequently examined using growth models, often estimated using linear mixed models. However, thus far, generative modeling frameworks and multilevel models (Bryk & Raudenbush, 1992) have yet to be integrated. Here, we illustrate a means of estimating longitudinal changes in parameters from computational models in a single model. We demonstrate this model estimation first from a simulation of a single parameter reward learning model and use a real-world example of longitudinal changes in the Iowa Gambling Task (IGT; Bechara et al., 1994; Cauffman et al., 2010) across a five-wave longitudinal study.

Many areas of research rely on behavioral assessments to assess decision making, reward sensitivity, and response biases. Historically, studies using these assessments rely on gross metrics of behavioral performance. For these measures, a critical challenge is that individual behavioral outputs are produced via multiple psychological processes (Wiecki et al., 2015), and behavioral measures frequently represent gross characterizations of the many competing and complementary processes that give rise to the observable behavior. While conventional scoring procedures are unable to discriminate between these processes, more recently developed computational models of behavior are well-positioned to discriminate between individual psychological processes, yielding enhanced specificity in behavioral metrics as well as improved task psychometrics. For example, [Nate’s initial IGT modeling paper?]

### Longitudinal Research

Longitudinal research takes many forms, with foci on either or both mean-level and rank-order stability. In studies of test-retest reliability, many studies examine how task performance is consistent across two waves of assessments. [Maybe briefly note the use in computational modeling with our studies and/or others?]. However, in these contexts, the only means of evaluating change involves a simple difference or change from one occasion to another (Ployhart and MacKenzie (2014).

In the context of longitudinal development, course, or intervention outcome, studies frequently employ more than two assessments, which provides flexibility in the modeling of change across time, including changes in mean-level and rank-order stability in the same model. Some methods, such as repeated measures analysis of variance (RM-ANOVA), offers a means of testing mean-level differences between assessments. RM-ANOVA is typically implemented by estimating simple mean-level differences and is unable to accommodate missing data, without the use of other methods (e.g., multiple imputation). Other methods, including multilevel models (MLMs) and latent growth curve models (LGCMs), provide additional flexibility for considering underlying trajectories of change that explain the mean-level changes in outcomes. Despite their differences in data organizational structures, the estimation of MLMs and LGCMs are identical, when requisite constraints are applied. The trajectories are characterized by point estimates of starting points (i.e., intercepts) and rates of change (i.e., slopes), as well as random effects reflecting individual differences in intercepts and slopes. With behavioral measures, studies have used summary behavioral metrics as variables at each timepoint. As noted above, these indices may conflate multiple processes leading to the behaviors.

Some attempts have been made to estimate longitudinal trajectories of computational models. Researchers examining longitudinal changes in behavioral processes do so in two-stage approaches. First, a behavioral model is fit to the data at each timepoint separately, and then second, the longitudinal model is fit to the parameters from the behavioral model. Such an approach has yielded important insights so far regarding how some behavioral processes develop across time. For example, Klein et al. (2022) used a hyperbolic discounting model and a multilevel model to examine developmental changes in delay discounting across time, finding that the degree of delay discounting tends to decrease rapidly early in childhood and begins to level off in mid-to-late adolescence. We can improve upon these methods to provide further insights regarding longitudinal changes in behavioral processes by embedding the behavioral model within the longitudinal model to avoid having to use two-stage approaches. Such a method could improve estimates of how computationally-derived parameters change over time because we can use information derived from all participants and all timepoints to inform estimates of different individuals and at different timepoints.

#### 1.1.2 Benefits of longitudinal designs

* 1. Examine change at both group and individual level
  2. Establish sequence of events (i.e., what predicts what)

#### 1.1.3 Drawbacks to longitudinal designs

* 1. Expensive & difficult
  2. Random assignment of variables is uncommon; thus, cannot establish causation
  3. Sequence effects may bias results

### 1.2 Longitudinal Modeling Methods

#### 1.2.1 RM ANOVAs

#### 1.2.2 Multilevel modeling

#### 1.2.3 Latent growth curve modeling

### 1.3 Current study

1. Prior longitudinal methods rely only on general linear model (i.e., cannot structure theoretical model to capture growth within the model)
   1. Good place to put in McElreath quote about GLM – something like “definitely wrong but hard to beat”
   2. To incorporate theoretical model, typically have to use two-stage approach
2. Here, we show how to incorporate growth-related parameters in computational models so that our theoretical model can capture growth
   1. Benefits
      1. Propagate uncertainty across multiple levels of analysis which improves inferences
      2. Allows us to use theoretical models to examine growth instead of summary statistics
         1. i.e., better aligns statistical model with theoretical model

## 2 Simple Longitudinal RL Model

To illustrate the longitudinal computational modeling framework, we begin with a simulated example of how to construct such a model. We first constructed a hypothetical task modeled after the Iowa Gambling Task, a task for which computational models are frequently employed to understand. For the hypothetical task, participants are presented with two options with the same expected values but with different outcomes and different outcome probabilities across 60 total trials. Choices on one option yield either $75 or $25 with equal probability (i.e., *P*($75) = *P*($25) = .5), resulting in $50 on average across trials. Choices on the other option yield either $80 or $40 with a .75 and .25 probability, respectively, also resulting in $50 on average across trials. Next, we built a one-parameter longitudinal reinforcement learning model to simulate data for the task across four conditions. Choices within the task are assumed to be drawn from a Bernoulli distribution, such that

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| --- | --- | --- |
|  |  | Equation 1 |

where *Yi,s*(*t*) is the choice for either option 1 (*Y* = 0) and option 2 (*Y* = 1) on trial *t* by participant *i* on session *s*, and *V0,i,s*(*t*) and *V1,i,s*(*t*) are the expected values associated with choosing option 1 or 2, updated from trial to trial according to the following function:

|  |  |  |
| --- | --- | --- |
|  |  | Equation 2 |

where *A* is a free parameter describing learning rate for both options, and *x*(*t*) is the amount of the outcome on trial *t*. Equations 1 and 2 represent a simple reinforcement learning model describing how gains on both options affect choices for those options.

|  |  |  |
| --- | --- | --- |
|  | No cor | Moderate cor |
| No effect | *rtime* = 0, *d* = 0 | *rtime* = .3, *d* = 0 |
| Moderate effect | *rtime* = 0, *d* = .5 | *rtime* = .3, *d* = .5 |

The four conditions represent parametric combinations of two levels of test-retest reliability, unreliable (i.e., *r* = 0) and moderate reliability (i.e., *r* = .3) and two levels of longitudinal change, no change (i.e., *d* = 0) and moderate change (i.e., *d* = .5; Cohen, 2016). Finally, after simulating data, we examined how well parameters could be recovered using more conventional (e.g., two-stage) approaches for analyzing longitudinal data using computational modeling.

## 3 Longitudinal Model of Iowa Gambling Task

## 4 Discussion

### 4.1 Benefits of this approach

### 4.2 Drawbacks of this approach

# References

# Tables

# Figures